19. $\lim_{x \to 0} \frac{|x|}{x}$ is

(A) 0 **(B)** nonexistent **(C)** 1 **(D)** −1 **(E)** none of these

20. $\lim_{x \to \infty} x \sin \frac{1}{x}$ is

(A) 0 **(B)** ∞ **(C)** nonexistent **(D)** −1 **(E)** 1

21. $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi - x}$ is

(A) 1 **(B)** 0 **(C)** ∞ **(D)** nonexistent **(E)** none of these

22. Let $f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ 4 & \text{if } x = 1. \end{cases}$

Which of the following statements is (are) true?

I. $\lim_{x \to 1} f(x)$ exists
II. $f(1)$ exists
III. f is continuous at $x = 1$

(A) I only **(B)** II only **(C)** I and II
(D) none of them **(E)** all of them

23. If $\begin{cases} f(x) = \frac{x^2 - x}{2x} \text{ for } x \neq 0, \\ f(0) = k, \end{cases}$

and if f is continuous at $x = 0$, then $k =$

(A) −1 **(B)** $-\frac{1}{2}$ **(C)** 0 **(D)** $\frac{1}{2}$ **(E)** 1

24. Suppose $\begin{cases} f(x) = \frac{3x(x-1)}{x^2 - 3x + 2} \text{ for } x \neq 1, 2, \\ f(1) = -3, \\ f(2) = 4. \end{cases}$

Then $f(x)$ is continuous

(A) except at $x = 1$ **(B)** except at $x = 2$ **(C)** except at $x = 1$ or 2
(D) except at $x = 0$, 1, or 2 **(E)** at each real number

25. The graph of $f(x) = \frac{4}{x^2 - 1}$ has

(A) one vertical asymptote, at $x = 1$
(B) the y-axis as vertical asymptote
(C) the x-axis as horizontal asymptote and $x = \pm 1$ as vertical asymptotes
(D) two vertical asymptotes, at $x = \pm 1$, but no horizontal asymptote
(E) no asymptote

26. The graph of $y = \frac{2x^2+2x+3}{4x^2-4x}$ has

(A) a horizontal asymptote at $y = +\frac{1}{2}$ but no vertical asymptote
(B) no horizontal asymptote but two vertical asymptotes, at $x = 0$ and $x = 1$
(C) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = 0$ and $x = 1$
(D) a horizontal asymptote at $x = 2$ but no vertical asymptote
(E) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = \pm 1$

27. Let $f(x) = \begin{cases} \frac{x^2+x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$.

Which of the following statements is (are) true?

I. $f(0)$ exists
II. $\lim_{x\to 0} f(x)$ exists
III. f is continuous at $x = 0$

(A) I only **(B)** II only **(C)** I and II only
(D) all of them **(E)** none of them

Part B. Directions: Some of the following questions require the use of a graphing calculator.

28. If $[x]$ is the greatest integer not greater than x, then $\lim_{x\to 1/2}[x]$ is

(A) $\frac{1}{2}$ **(B)** 1 **(C)** nonexistent **(D)** 0 **(E)** none of these

29. (With the same notation) $\lim_{x\to -2}[x]$ is

(A) -3 **(B)** -2 **(C)** -1 **(D)** 0 **(E)** none of these

30. $\lim_{x\to\infty} \sin x$

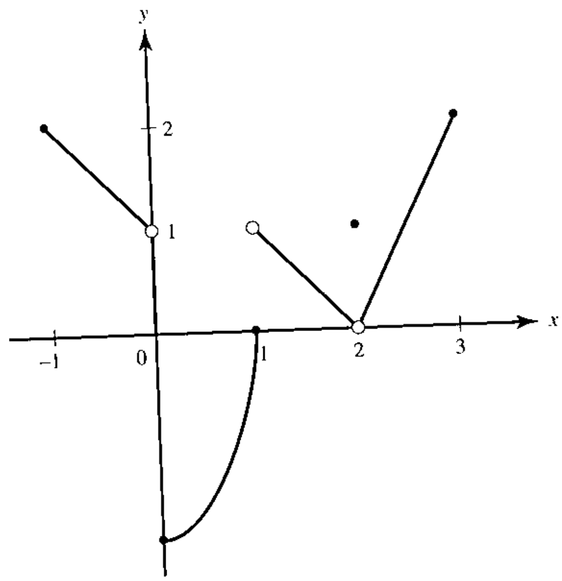
(A) is -1 **(B)** is infinity **(C)** oscillates between -1 and 1
(D) is zero **(E)** does not exist

31. The function $f(x) = \begin{cases} x^2/x & (x \neq 0) \\ 0 & (x = 0) \end{cases}$

(A) is continuous everywhere
(B) is continuous except at $x = 0$
(C) has a removable discontinuity at $x = 0$
(D) has an infinite discontinuity at $x = 0$
(E) has $x = 0$ as a vertical asymptote

Questions 32–36 are based on the function f shown in the graph and defined below:

$$f(x)=\begin{cases} 1-x & (-1 \leqslant x < 0) \\ 2x^2-2 & (0 \leqslant x \leqslant 1) \\ -x+2 & (1 < x < 2) \\ 1 & (x=2) \\ 2x-4 & (2 < x \leqslant 3) \end{cases}$$



32. $\lim\limits_{x \to 2} f(x)$

(A) equals 0 **(B)** equals 1 **(C)** equals 2
(D) does not exist **(E)** none of these

33. The function f is defined on $[-1,3]$

(A) if $x \neq 0$ **(B)** if $x \neq 1$ **(C)** if $x \neq 2$
(D) if $x \neq 3$ **(E)** at each x in $[-1,3]$

34. The function f has a removable discontinuity at

(A) $x = 0$ **(B)** $x = 1$ **(C)** $x = 2$ **(D)** $x = 3$ **(E)** none of these

35. On which of the following intervals is f continuous?

(A) $-1 \leqslant x \leqslant 0$ **(B)** $0 < x < 1$ **(C)** $1 \leqslant x \leqslant 2$
(D) $2 \leqslant x \leqslant 3$ **(E)** none of these

36. The function f has a jump discontinuity at

(A) $x = -1$ **(B)** $x = 1$ **(C)** $x = 2$
(D) $x = 3$ **(E)** none of these

CHALLENGE

37. $\lim\limits_{x \to 0} \sqrt{3 + \arctan \frac{1}{x}}$ is

(A) $-\infty$ **(B)** $\sqrt{3 - \frac{\pi}{2}}$ **(C)** $\sqrt{3 + \frac{\pi}{2}}$

(D) ∞ **(E)** none of these

38. Suppose $\lim_{x\to -3^-} f(x) = -1$, $\lim_{x\to -3^+} f(x) = -1$, and $f(-3)$ is not defined. Which of the following statements is (are) true?

I. $\lim_{x\to -3} f(x) = -1$.
II. f is continuous everywhere except at $x = -3$.
III. f has a removable discontinuity at $x = -3$.

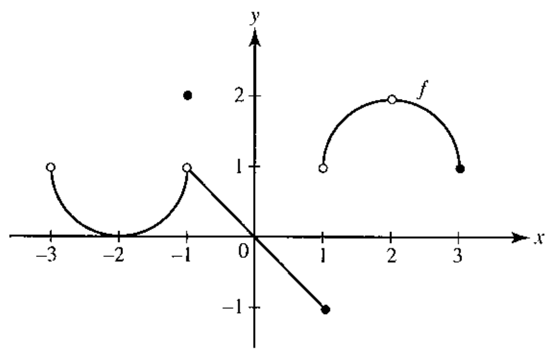
(A) None of them **(B)** I only **(C)** III only
(D) I and III only **(E)** All of them

39. If $y = \dfrac{1}{2 + 10^{\frac{1}{x}}}$, then $\lim_{x\to 0} y$ is

CHALLENGE

(A) 0 **(B)** $\dfrac{1}{12}$ **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{1}{3}$ **(E)** nonexistent

Questions 40–42 are based on the function f shown in the graph.



40. For what value(s) of a is it true that $\lim_{x\to a} f(x)$ exists and $f(a)$ exists, but $\lim_{x\to a} f(x) \neq f(a)$? It is possible that $a =$

(A) −1 only **(B)** 1 only **(C)** 2 only
(D) −1 or 1 only **(E)** −1 or 2 only

41. $\lim_{x\to a} f(x)$ does not exist for $a =$

(A) −1 only **(B)** 1 only **(C)** 2 only
(D) 1 and 2 only **(E)** −1, 1, and 2

42. Which statements about limits at $x = 1$ are true?

I. $\lim_{x\to 1^-} f(x)$ exists.
II. $\lim_{x\to 1^+} f(x)$ exists.
III. $\lim_{x\to 1} f(x)$ exists.

(A) none of I, II, or III **(B)** I only **(C)** II only
(D) I and II only **(E)** I, II, and III